DOCUMENT RESUME

ED 050 962 SE 011 251

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TITLE	Preferences for Modes of Expression in Mathematics.
INSTITUTION	Stanford Univ., Calif. School of Education.
PUB DATE	May 67
NOTE	25p.
EDRS PRICE	EDRS Price MF-$0.65 HC-$3.29
DESCRIPTORS	Graphs, Instruction, *Learning, *Mathematical Vocabulary, *Mathematics Education, *Research, Secondary School Mathematics, *Symbols (Mathematics)

ABSTRACT

This is a preliminary report of a project that is attempting to extend psychological research on stylistic differences in cognition to the study of teaching. This portion of the project measures students' preferences for three modes of mathematical expression; symbolic, verbal, and graphic. One hundred and fifteen junior high school students took a test written to measure preferences for the three modes of mathematical expression. In addition, student scores on the Iowa Test of Basic Skills, Form 1, were obtained. An item analysis was made and correlations made between total scores on each of the three scales and the nine scores from the Iowa Test. The results were that students preferred a symbolic mode of representation and that the verbal and graphic scales formed positively skewed distributions while the symbolic scale formed a relatively symetric though bimodal distribution. Other results are summarized in tabular form. Several suggestions for further experimentation are given. (Author/CT)

ED050962

STANFORD CENTER FOR RESEARCH AND DEVELOPMENT IN TEACHING

Research Memorandum No. 7

PREFERENCES FOR MODES OF EXPRESSION IN MATHEMATICS

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School of Education
Stanford University
Stanford, California

May 1967

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SE 011 325

This is a preliminary report of a project that is attempting to extend psychological research on stylistic differences in cognition to the study of teaching.[1] The emphasis of prior research on cognitive styles has been toward the "learner" half of the hypothetical "teacher-learner" interactive process. In this research, it is planned to measure cognitive preferences on both sides of this process and to study the interactive relations between the two.

Our first hypothesis is that people (both students and teachers) differ reliably in their preferences for modes of mathematical expression. If such cognitive preferences exist and if an instrument can be developed to measure them, the relations of these preferences to educational effectiveness may be studied. For example:

1. Are preferences for modes of expression of math teachers related to teaching success?
2. Does correspondence between teacher and student preferences facilitate instruction?
3. Should teachers be trained to use different modes of expression with different kinds of students and with different mathematical topics?
4. Are preferences for modes of mathematical expression modified by instruction: (a) Does institute training, technical skills training, etc., alter the teachers' preferences? (b) Do different practices of teachers alter the students' preferences for different modes of expression in mathematics?
5. What are the relations between preference for modes of mathematical expression and other aptitude variables; (a) for students, (b) for teachers?

The development of an instrument to measure students' and teachers' preferences for modes of mathematical expression has been undertaken. Three a priori modes of expression were specified: verbal, symbolic and graphic. Thirty items were written for pretesting. (Pretest form in Appendix A.) Each item requires the respondent to choose from among the three modes of expressing a single mathematical idea.

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[1] The interest and assistance of the School of Mathematics' Study Group are gratefully acknowledged.

In March 1967, the trial form of the instrument was pretested. The sample consisted of 115 seventh-grade students (69 males - 46 females) from a junior high school in Fremont, California.[2] Four classes were used, yielding a representative cross-section of the school's mathematics achievement levels. The test was administered by SCRDT personnel during the regular mathematics period. Although an administration time of forty minutes (the length of the regular class) was allowed, most students completed the test in approximately twenty minutes. In addition, student scores on the Iowa Test of Basic Skills, Form 1, given in October 1966, were obtained from the school files.

Table 1 presents the summary statistics from the pretest. In general, the symbolic option was most often preferred. The various reliability indexes reported are not directly interpretable because of the ipsative nature of the scales.

As Figure 1 shows, the Verbal and Graphic scales form positively skewed distributions while the Symbolic scale forms a relatively symmetric though bimodal distribution.

Results of item analyses are given in Table 2. The biserial $\underline{r}$ and proportion-choosing are shown for each item in relation to each of the three scales.

Table 3 gives the correlations between total scores for each of the three scales and nine Iowa Test of Basic Skills (Form I) scores. Again, the interpretation of these correlations is confused by the logical interdependence of the preference scales.

The pretest indicated that (a) the 30-item test can be comfortably administered in a classroom period, (b) the items function as intended and appear to discriminate, and (c) a usable balance among preference scores is produced by the test.

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[2] Grateful acknowledgment is made to Donald Edgar, SCRDT assistant, for his help in administering the mathematics Cognitive Preference Test and collecting other data on the students.

Four tasks are suggested: (a) a revision of the test directions that makes the basis for choice of option clearer, (b) study of the psychometric problem arising from the ipsative scoring, (c) trial administration to a sample of teachers to check the test's appropriateness for that group, and (d) trial administration to samples of students in other grades to check stability of scores across chronological age and achievement levels.

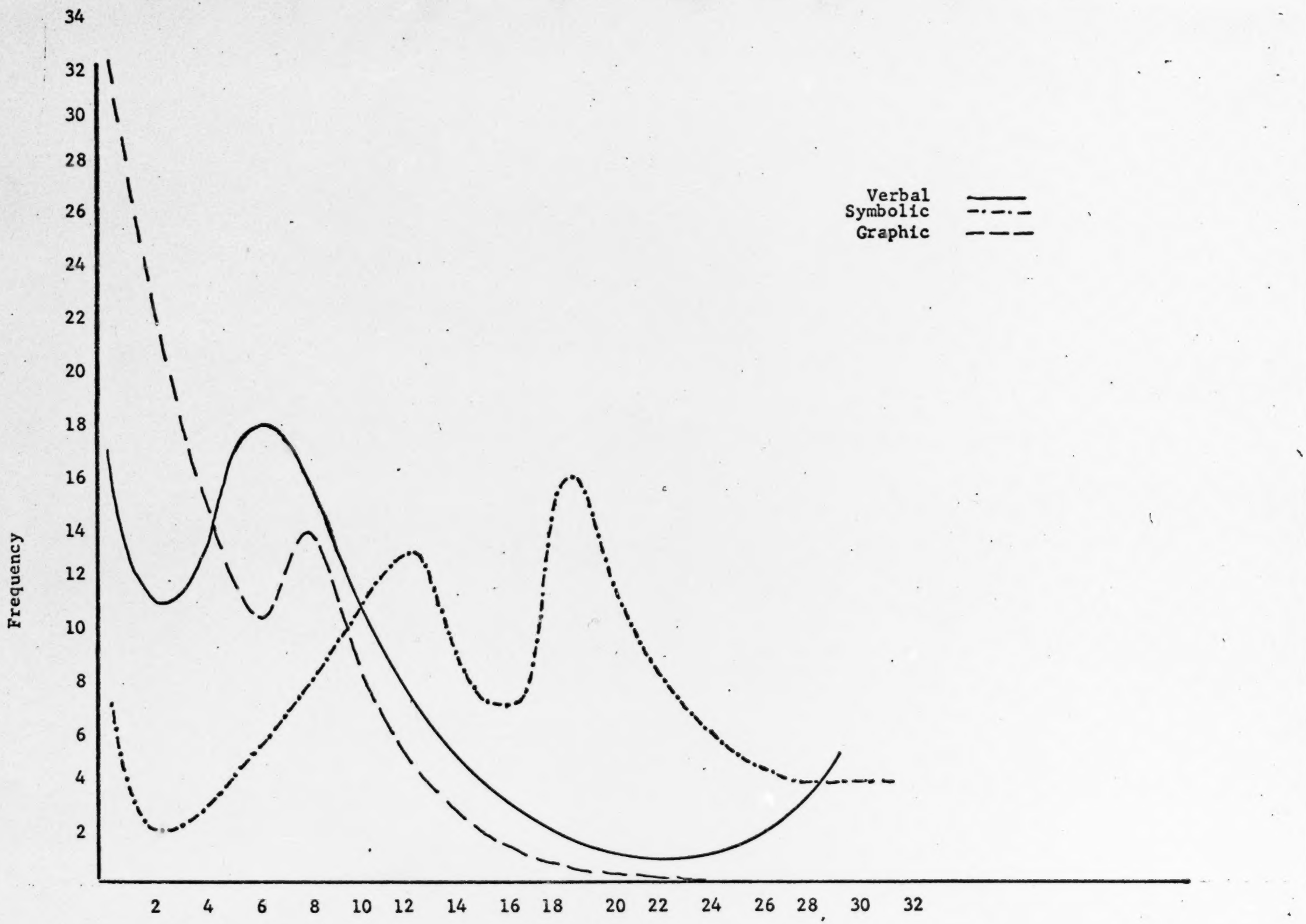


Figure 1

Frequency Distribution of Cognitive Preference Scores

Table 1

Summary Statistics

Cognitive Preference Pretest

	Mean	Standard Deviation	Variance	Skewness	Kurtosis	Pearson r	Cronbach's Alpha	Corrected Split Half	Guttman L4	Median
Verbal	8.75	7.20	51.87	1.09	0.89	0.87	0.91	0.93	0.93	7.00
Symbolic	15.73	7.76	60.29	-0.14	-0.64	0.82	0.91	0.90	0.90	17.00
Graphic	5.47	5.50	30.23	1.29	1.43	0.82	0.89	0.90	0.90	4.00

Table 2

Item Analysis of Cognitive Preference Pretest

Item	Verbal		Symbolic		Graphic	
	r_{bis}	P *	r_{bis}	P	r_{bis}	P
1	.575	.235	.647	.617	.600	.148
2	.374	.243	.500	.478	.584	.278
3	1.115	.114	.871	.816	.790	.070
4	.740	.237	.568	.561	.658	.202
5	.791	.261	.805	.643	.744	.096
6	.680	.287	.579	.470	.554	.243
7	.626	.287	.560	.565	.704	.148
8	.754	.365	.709	.365	.680	.270
9	.571	.365	.619	.330	.614	.304
10	.696	.400	.642	.426	.873	.174
11	.599	.377	.612	.360	.488	.263
12	.814	.316	.652	.474	.725	.210
13	.657	.278	.537	.287	.327	.435
14	.673	.130	.884	.765	.709	.104
15	.647	.348	.674	.435	.814	.217
16	.557	.439	.459	.412	.824	.149
17	.657	.298	.726	.491	.995	.210
18	.646	.304	.664	.539	.767	.156
19	.689	.278	.475	.617	.838	.104
20	.715	.287	.774	.556	.854	.156
21	.646	.322	.699	.470	.830	.209
22	.840	.330	.695	.548	.713	.122
23	.889	.130	.715	.713	.552	.156
24	.614	.339	.621	.348	.656	.313
25	.676	.461	.651	.365	.873	.174
26	.558	.313	.553	.539	.696	.148
27	.762	.261	.759	.626	.587	.113
28	.688	.296	.576	.643	.525	.061
29	.888	.287	.730	.556	.818	.156
30	.718	.174	.711	.739	.410	.087

* P = proportion of people choosing item, corrected for non-response

Table 3

Correlation Coefficients of Cognitive Preference Pretest

Iowa Tests of Basic Skills	Verbal	Symbolic	Graphic
Vocab	-.182	.313**	-.198*
Reading	-.125	.314**	-.271**
Tot Lang	-.157	.349**	-.278**
Graphs	-.288**	.340**	-.105
Tot Work Study	-.243*	.367**	-.195*
Arith Concepts	-.270**	.416**	-.229*
Arith Probs	-.123	.270**	-.211*
Tot Arith	-.213*	.359**	-.221*
Composite	-.187	.356**	-.250**

* Significant at the .05 level
** Significant at the .01 level

Form SJ

COGNITIVE PREFERENCE TEST

MATHEMATICS

Name_____ School_____

Directions

Mathematical ideas can be expressed in different ways. In this test you are to choose among three correct ways of expressing the same mathematical idea.

Each test item consists of three choices dealing with the same topic in junior high school mathematics. Read all three of the choices carefully to find the mathematical idea that they have in common.

If you were learning about this mathematical topic, in which of the three ways would you prefer your teacher to explain the idea to you? Mark the corresponding space on the answer sheet.

You may find that more than one choice for each item appeals to you. However, select only one choice for each item. Be sure to answer every question, even though the decision may be difficult to make.

A sample item is given on the next page. . .

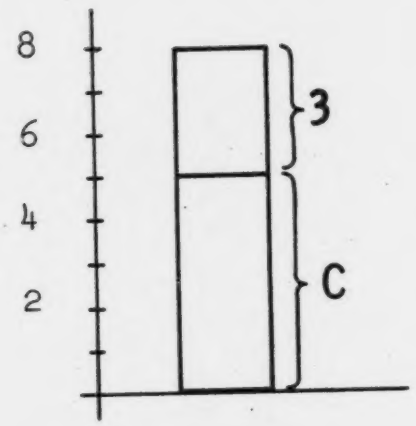
Sample Item

150:

a) A number added to three equals eight.

b) $C + 3 = 8$

c)



All three choices are correct ways of expressing the same mathematical idea. In which of these three ways would you prefer your teacher to explain the idea? Mark the corresponding space on the answer sheet.

NOTE: For this sample item, blacken the space corresponding to your choice opposite number 150 on the answer sheet.

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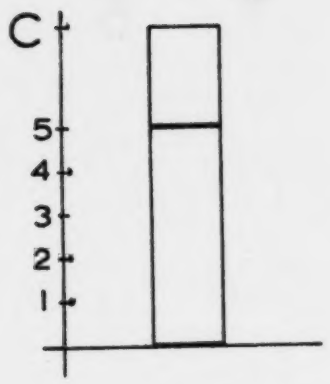
1.

a)



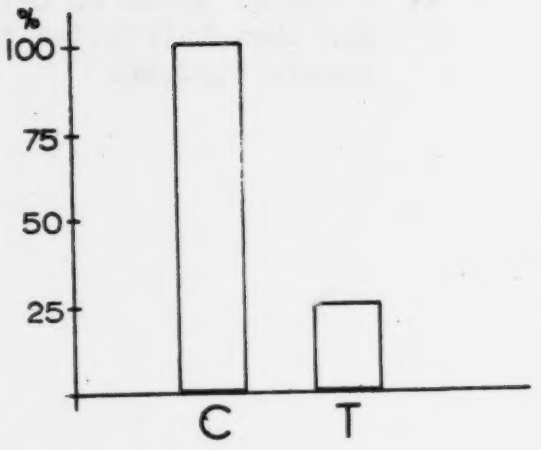
b) Five added to one number equals another number.

c) $5+T=C$

1. a b c

2.

a)



b) $\frac{25}{100} \times C = T$

c) Twenty-five per cent of one number equals another number.

2. a b c

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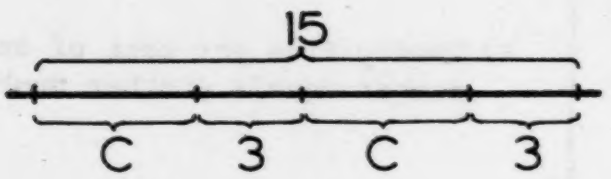
3.

a) A number added to three and then multiplied by two equals fifteen.

b) $2(C+3)=15$

c)

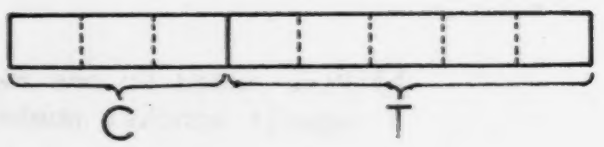


3. a b c

4.

a) $\frac{C}{T} = \frac{3}{5}$

b)



c) Two numbers are in the ratio of three to five.

4. a b c

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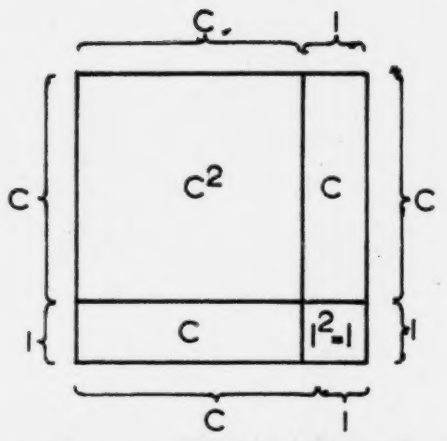
5.

a)



b) When a number plus one is multiplied by itself, it equals that number multiplied by itself plus two times that number plus one.

c) $(C+1)^2 = C^2+2C+1$

5. a b c

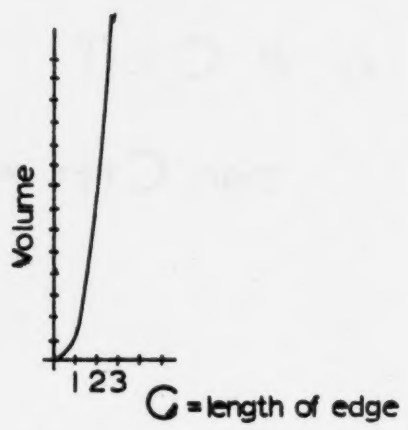
6.

a) The volume of a cube is equal to the length on edge multiplied by itself three times.

b) $\text{Volume} = C^3$

where the length of an edge of the cube =C

c)



6. a b c

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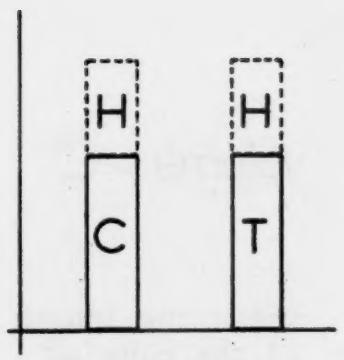
7.

a) If the same number is added to two equal numbers, the sums will be equal.

b)



c) If $C = T$

then $C+H = T+H$

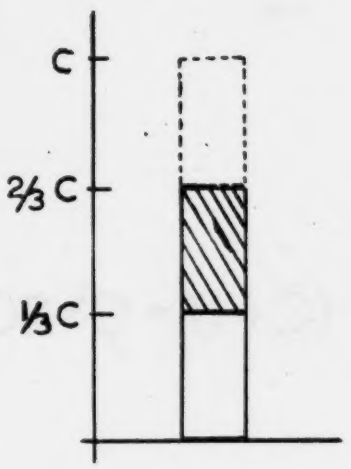
7. a b c

8.

a) $\frac{2}{3}H + 2 = \frac{1}{3}H$

b) Two-thirds of a number divided by two equals one-third of a number.

c)



8. a b c

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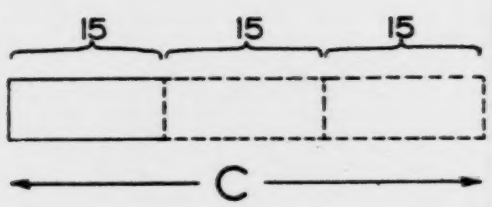
9.

a) One-third of some number is fifteen.

b)



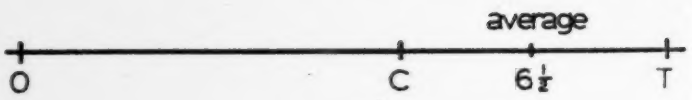
c) $\frac{1}{3}C = 15$

9. a b c

10.

a) $\frac{C+T}{2} = 6\frac{1}{2}$

b)



c) The average of two numbers is six and one-half.

10. a b c

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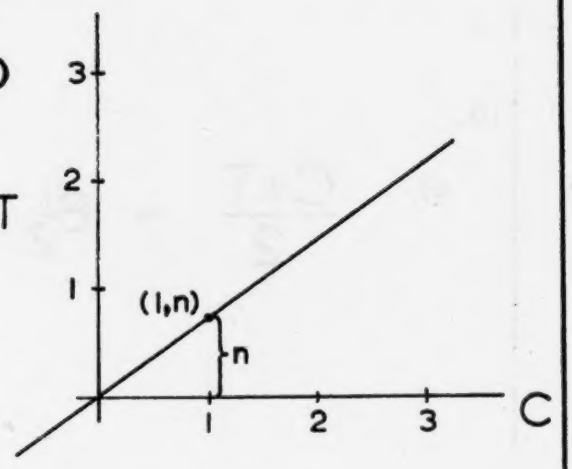
11.

a)



b) $T = nC$

c) Two numbers are directly proportional to one another.

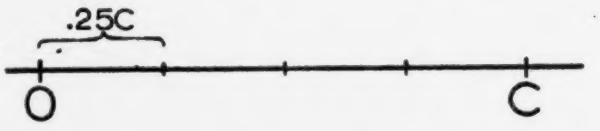
11. a b c

12.

a) A number divided by four equals twenty-five hundredths of that number.

b) $C \div 4 = .25C$

c)



12. a b c

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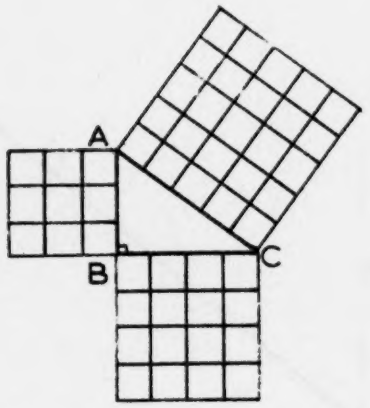
13.

a) For any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

b) $\overline{AC}^2 = \overline{BC}^2 + \overline{AB}^2$

where B is the right angle of ΔABC

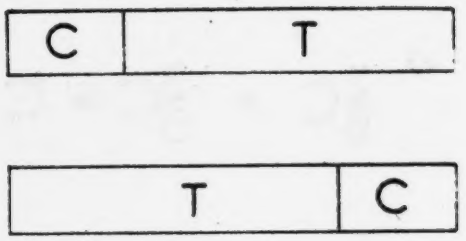
c)



13. a b c

14.

a)



b) For any two numbers, the sum of the first number added to the second number equals the sum of the second number added to the first number.

c) $C+T = T+C$

14. a b c

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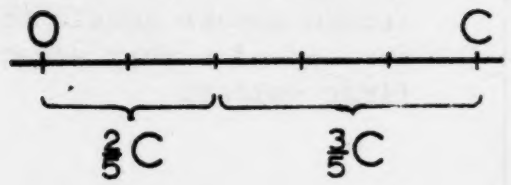
15.

a) $\frac{2}{5}C + \frac{3}{5}C = C$

b)



c) Two-fifths of a number added to three-fifths of a number equals the whole number.

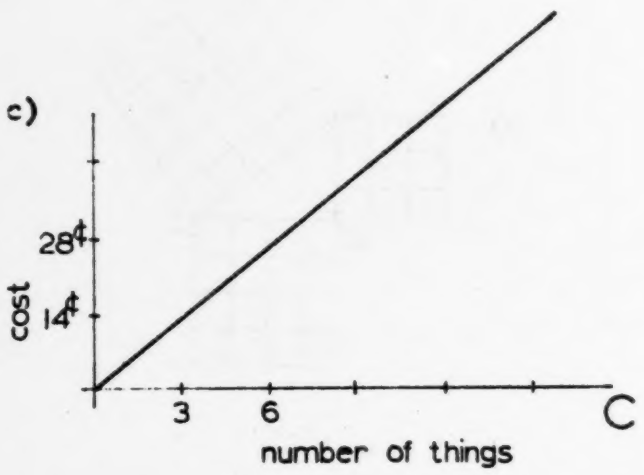
15. a b c

16.

a) $T \times \frac{14}{3}¢ = \text{Cost}$

b) If three things cost fourteen cents, then any number of those things will cost that number multiplied by fourteen cents and divided by three.

c)



16. a b c

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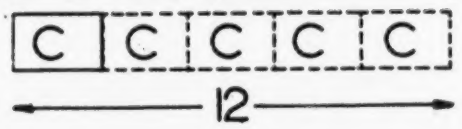
17.

a) $5C = 12$

b)



c) Five times some number equals twelve.

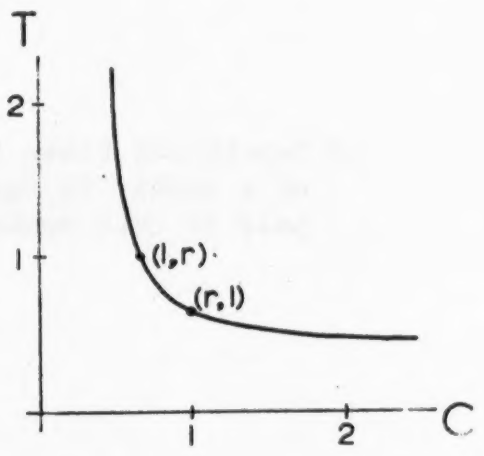
17. a b c

18.

a) $C \times T = r$

b) Two numbers are inversely proportional to one another.

c)



18. a b c

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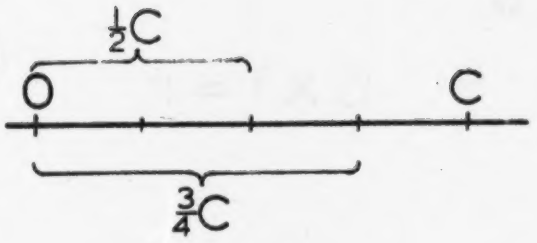
19.

a)



b) $\frac{2}{3} \times \frac{3}{4}C = \frac{1}{2}C$

c) Two-thirds times three-fourths of a number is equal to one-half of that number.

19. a b c

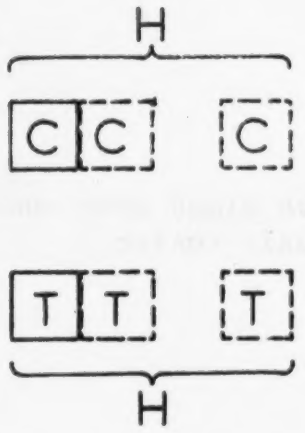
20.

a) If $C = T$

then $C \times H = T \times H$

b) If two equal numbers are multiplied by the same number, the products will be equal.

c)



20. a b c

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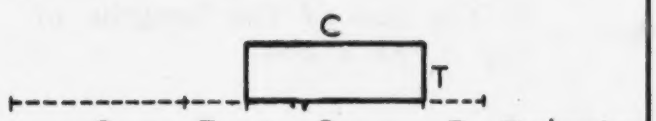
21.

a) Perimeter $= 2(C+T)$

where C = length of rectangle
T = width of rectangle

b)

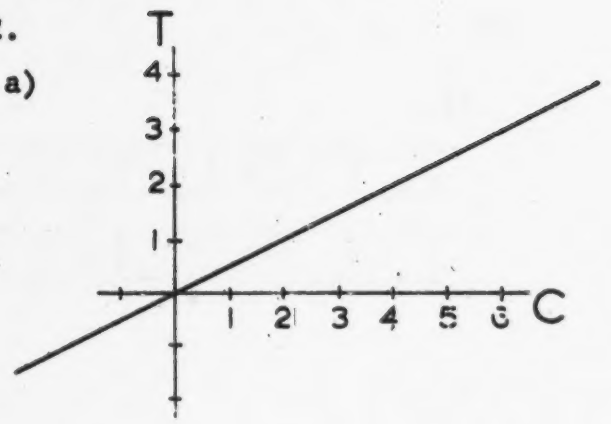


c) The perimeter of a rectangle is twice the sum of the width of the rectangle and the length of the rectangle.

21. a b c

22.

a)



b) $T = \frac{1}{2}C$

c) One number is equal to one-half of another number.

22. a b c

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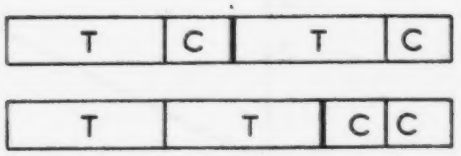
23.

a)



b)

$$2(T+C) = 2 \times T + 2 \times C$$

c) **Twice the sum of two numbers is equal to two times the first number plus two times the second number.**

23. a b c

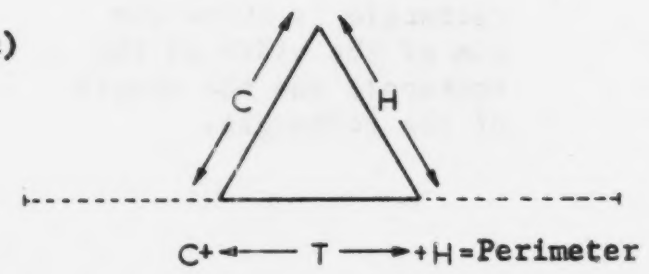
24.

a) **Perimeter** $= C+T+H$

where triangle has sides of length C, T, H.

b) **The perimeter of a triangle is the sum of the lengths of the three sides.**

c)



24. a b c

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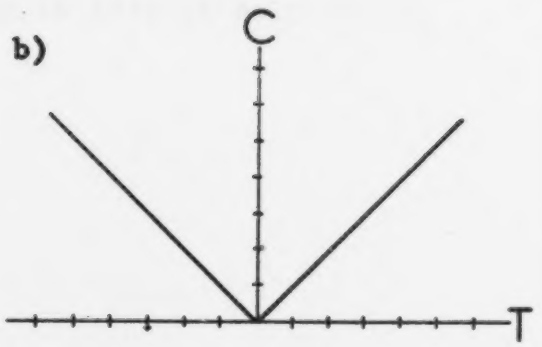
25.

a) $C = |T|$

b)



c) One number is equal to the absolute value of another number.

25. a b c

26.

a) Area = $C \times T$

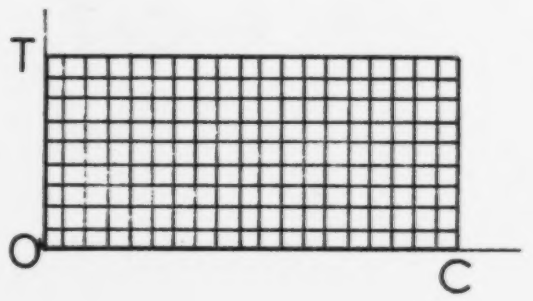
where

C = length of rectangle

T = width of rectangle

b) The area of a rectangle equals its width multiplied by its length.

c)



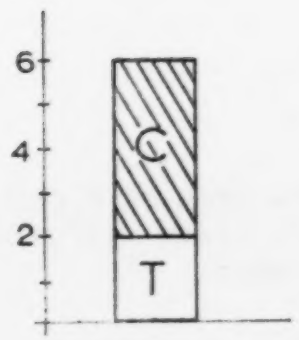
26. a b c

GO ON TO NEXT PAGE

27.

a) Six minus one number equals another number.

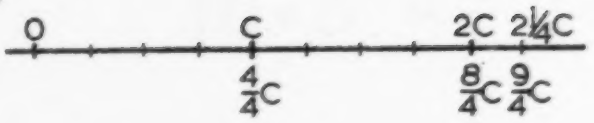
b)



c) $6 - C = T$

28.

a)



b) Two and one-fourth times a number equals nine-fourths of that number.

c) $2\frac{1}{4}C = \frac{9}{4}C$

27. a b c

28. a b c

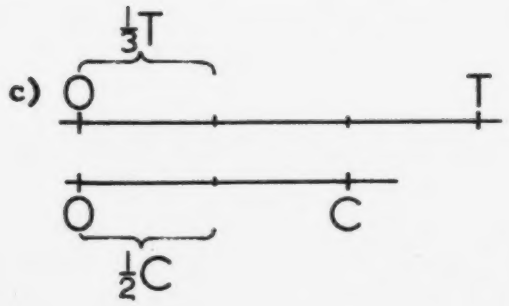
GO ON TO NEXT PAGE

29.

a) One-third of one number is equal to one-half of another number.

b) $\frac{1}{3}T = \frac{1}{2}C$

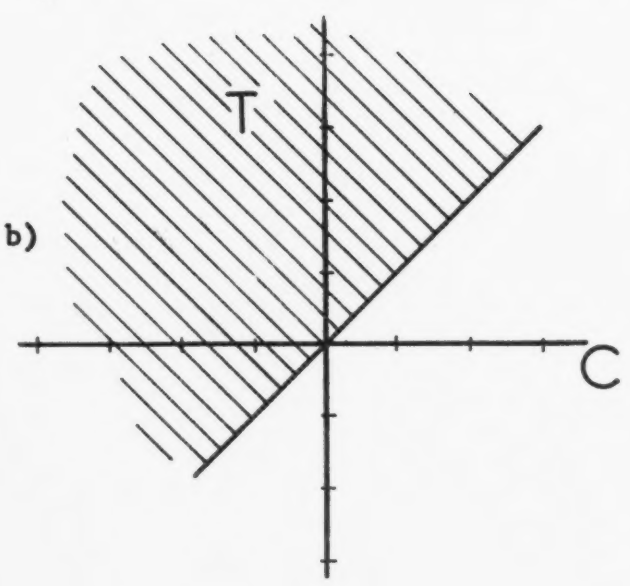
c)



30.

a) $T > C$

b)



c) One number is greater than another number.

29. a b c

30. a b c

THIS IS THE END OF THE TEST.

If you still have some time, check to see that you have not left out any items, but <u>do not change any choices you have made</u>.